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THE

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APRIL, 1860.

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J. D. RUNKLE, A.M., A.A.S.

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THE

MATHEMATICAL MONTHLY.

Vol. II. . . . APRIL, 1860. . . . No. VII.

PRIZE PROBLEMS FOR STUDENTS.

I. Prove that an arithmetic mean is greater than a geometric.

II. Let three bodies with velocities V, V', V'', move uniformly in the same direction, in the circumference of a circle. Required the time of their conjunction, supposing them to quit a given point at the same time.

III. The diameter of a circle inscribed in the quadrant of a second circle is equal to the side of the regular octagon circumscribed about the second circle. — Communicated by Prof. Henry H. White.

IV. Required the locus of the centres of the circles inscribed within all the right-angled triangles which can be inscribed in a given semicircle. — Communicated by Prof. Henry H. White.

V. From a box containing a very large number of white and black balls, of each an equal number, three balls are taken at random and placed in a bag without being seen. A takes a ball at random from the bag, observes its color, and replaces it four times in succession. The ball was white on each of the four drawings. What are the respective probabilities that the bag contains 1, 2, or 3 white balls?

Solutions of these problems must be received by June 1, 1860.

REPORT OF THE JUDGES UPON THE SOLUTIONS OF THE PRIZE PROBLEMS IN No. IV., Vol. II.

THE first Prize is awarded to W. F. OSBORNE, of the Wesleyan University, Middletown, Ct.

The second Prize is awarded to M. K. BOSWORTH, of the Sophomore class, Marietta College, Ohio.

The third Prize is awarded to E. S. STEARNS, of Chester Institute, Chester, Morris Co., N. J.

PRIZE SOLUTION OF PROBLEM II.

By E. S. STEARNS, Chester Institute, N. J.

Show that $\frac{2+\sqrt{3}}{\sqrt{2+\sqrt{(2+\sqrt{3})}}}+\frac{2-\sqrt{3}}{\sqrt{2-\sqrt{(2-\sqrt{3})}}}=\sqrt{2}$.

We have by the formula

$$\sqrt{a \pm \sqrt{b}}=\sqrt{\frac{a+c}{2}} \pm \sqrt{\frac{a-c}{2}},$$

in which $c=\sqrt{a^2-b}$ (see algebra),

$$\sqrt{2+\sqrt{3}}=\frac{\sqrt{3}+1}{\sqrt{2}}, \text{ and } -\sqrt{2-\sqrt{3}}=\frac{1-\sqrt{3}}{\sqrt{2}}.$$

$$\therefore \quad \sqrt{2+\sqrt{2+\sqrt{3}}}=\frac{3+\sqrt{3}}{\sqrt{2}} \text{ and } \sqrt{2-\sqrt{2-\sqrt{3}}}=\frac{3-\sqrt{3}}{\sqrt{2}}.$$

Therefore the given fractions become

$$\frac{(2+\sqrt{3})\sqrt{2}}{3+\sqrt{3}}+\frac{(2-\sqrt{3})\sqrt{2}}{3-\sqrt{3}}=\frac{3\sqrt{2}+\sqrt{6}}{6}+\frac{3\sqrt{2}-\sqrt{6}}{6}=\sqrt{2}.$$

PRIZE SOLUTION OF PROBLEM III.

By W. F. OSBORNE, Wesleyan University, Middletown, Ct.

Find the roots of the equation $x^3-6x=4$, by trigonometry.

By trigonometry, $\sin 3\theta = 3\sin\theta - 4\sin^3\theta$; whence

$$(1) \qquad \sin^3\theta - \tfrac{3}{4}\sin\theta + \tfrac{1}{4}\sin 3\theta = 0.$$

Substituting in the given equation $x=\varrho\sin\theta$, it becomes

(2) $$\sin^3\theta - \frac{6}{\rho^2}\sin\theta - \frac{4}{\rho^3} = 0;$$

in which any arbitrary value my be assigned to ρ. Hence, if we put $\frac{6}{\rho^2} = \frac{3}{4}$, or $\rho = \sqrt{8}$, by comparing (1) and (2) we shall have

$$\sin 3\theta = -\frac{16}{\rho^3} = -\frac{16}{8\sqrt{8}} = -\sqrt{\tfrac{1}{2}}. \qquad \therefore 3\theta = \left(2n + \frac{2 \pm 3}{4}\right)\pi.$$

Hence, $\sin\theta = \sin\left(\frac{2n}{3} + \frac{2 \pm 3}{12}\right)\pi$, from which all the different values of $\sin\theta$ may be obtained by making $n = 0$ or ± 1. Therefore the three values of x are

$$-\sqrt{8}\sin 15° = 1 - \sqrt{3}, \quad \sqrt{8}\cos 15° = 1 + \sqrt{3}, \quad -\sqrt{8}\cos 45° = -2.$$

Prize Solution of Problem IV.

By W. F. Osborne, Wesleyan University, Middletown, Ct.

Four persons, A, B, C, D, in order, beginning with A, cut a pack of cards, replacing them after each cut, on condition that the first who cuts a heart shall win. What are their respective probabilities of success?

The probability that a heart will be cut on the first trial is evidently $\frac{1}{4}$, and $\frac{3}{4}$ against it; on the second trial $\frac{1}{4} \times \frac{3}{4}$ for, and $\frac{3}{4} \times \frac{3}{4}$ against it; on the third trial $\frac{1}{4} \times \frac{3}{4} \times \frac{3}{4}$ for, and $\frac{3}{4} \times \frac{3}{4} \times \frac{3}{4}$ against it; and so on to the nth trial, on which the probability that a heart will be cut is $\frac{1}{4}\left(\frac{3}{4}\right)^{n-1}$, and $\left(\frac{3}{4}\right)^n$ against it.

But A has the first, fifth, ninth, &c. cuts; hence his probability of success is

$$\tfrac{1}{4}\left(1 + \left(\tfrac{3}{4}\right)^4 + \left(\tfrac{3}{4}\right)^8 + \text{\&c.}\right) = \tfrac{1}{4} \cdot \frac{1}{1 - \left(\frac{3}{4}\right)^4} = \tfrac{64}{175}.$$

B has the second, sixth, &c. cuts, and hence his probability of success is

$$\tfrac{1}{4} \cdot \tfrac{3}{4}\left(1 + \left(\tfrac{3}{4}\right)^4 + \left(\tfrac{3}{4}\right)^8 + \text{\&c.}\right) = \tfrac{1}{4} \cdot \tfrac{3}{4} \cdot \frac{1}{1 - \left(\frac{3}{4}\right)^4} = \tfrac{48}{175}.$$

In precisely the same manner C's and D's probabilities of success are

$$\tfrac{1}{4} \cdot \left(\tfrac{3}{4}\right)^2 \cdot \frac{1}{1 - \left(\frac{3}{4}\right)^4} = \tfrac{36}{175}, \qquad \tfrac{1}{4} \cdot \left(\tfrac{3}{4}\right)^3 \cdot \frac{1}{1 - \left(\frac{3}{4}\right)^4} = \tfrac{27}{175}.$$

Prize Solution of Problem V.

By Asher B. Evans, Madison University, Hamilton, N. Y.

The notation of Problem V. in the November number of the Monthly being retained, prove that in the plane

(1) $$\cot\tfrac{1}{2}A + \cot\tfrac{1}{2}B + \cot\tfrac{1}{2}C = \cot\tfrac{1}{2}A\cot\tfrac{1}{2}B\cot\tfrac{1}{2}C;$$

and in the sphere

(2) $$\cot\tfrac{1}{2}A + \cot\tfrac{1}{2}B + \cot\tfrac{1}{2}C = \cot\tfrac{1}{2}A\cot\tfrac{1}{2}B\cot\tfrac{1}{2}C\,\frac{\cos(\rho+\delta)\cos(\rho-\delta)}{\cos^2 r\cos^2\rho},$$

(3) $$\cot\tfrac{1}{2}A\,\frac{\cos\delta'}{\cos\rho'} + \cot\tfrac{1}{2}B\,\frac{\cos\delta''}{\cos\rho''} + \cot\tfrac{1}{2}C\,\frac{\cos\delta'''}{\cos\rho'''} = \cot\tfrac{1}{2}A\cot\tfrac{1}{2}B\cot\tfrac{1}{2}C\,\frac{\cos\delta}{\cos\rho}.$$

In the plane we have

$$\cot\tfrac{1}{2}A = \frac{s(s-a)}{n},\quad \cot\tfrac{1}{2}B = \frac{s(s-b)}{n},\quad \cot\tfrac{1}{2}C = \frac{s(s-c)}{n},$$

in which $s = \frac{1}{2}(a+b+c)$, $n^2 = s(s-a)(s-b)(s-c)$.

$$\therefore \cot\tfrac{1}{2}A + \cot\tfrac{1}{2}B + \cot\tfrac{1}{2}C = \frac{s^2}{n};$$

$$\therefore \cot\tfrac{1}{2}A\cot\tfrac{1}{2}B\cot\tfrac{1}{2}C = \frac{s^2(s-a)(s-b)(s-c)}{n^3} = \frac{s^2}{n};$$

therefore (1) is true. In the sphere

$$\frac{\cot\frac{1}{2}A}{\sin s} = \frac{\sin(s-a)}{n},\quad \frac{\cot\frac{1}{2}B}{\sin s} = \frac{\sin(s-b)}{n},\quad \frac{\cot\frac{1}{2}C}{\sin s} = \frac{\sin(s-c)}{n},$$

in which

$$s = \tfrac{1}{2}(a+b+c),\quad n^2 = \sin s\sin(s-a)\sin(s-b)\sin(s-c).$$

(4) $$\therefore \cot\tfrac{1}{2}A + \cot\tfrac{1}{2}B + \cot\tfrac{1}{2}C = \frac{\sin s\sin(s-a)\sin(s-b)\sin(s-c)}{n};$$

(5) $$\therefore \cot\tfrac{1}{2}A\cot\tfrac{1}{2}B\cot\tfrac{1}{2}C = \frac{\sin^2 s}{n}.$$

Also

$$\frac{\cos(\varrho+\delta)\cos(\varrho-\delta)}{\cos^2 r\cos^2\varrho} = \frac{\cos^2\delta - \sin^2\varrho}{\cos^2 r\cos^2\varrho} = \tan^2\varrho\left(\frac{\cos^2\delta}{\cos^2 r\sin^2\varrho} - 1 + \tan^2 r\right).$$

But we have (Chauvenet's Trig. Eqs. 307, 319, 332)

$$\tan r = \frac{2\sin\frac{1}{2}a\sin\frac{1}{2}b\sin\frac{1}{2}c}{n},\quad \tan\varrho = \frac{n}{\sin s},$$

$$\frac{\cos^2\delta}{\cos^2 r \sin^2\varrho} - 1 = \left(\frac{\sin s + 2\sin\frac{1}{2}a \sin\frac{1}{2}b \sin\frac{1}{2}c}{n}\right)^2.$$

$$\therefore \frac{\cos(\varrho+\delta)\cos(\varrho-\delta)}{\cos^2 r \cos^2\varrho} = \frac{\sin s + 4\sin\frac{1}{2}a \sin\frac{1}{2}b \sin\frac{1}{2}c}{\sin s}$$

$$(6) \qquad = \frac{\sin(s-a) + \sin(s-b) + \sin(s-c)}{\sin s}.$$

Combining (4), (5), (6), we get (2).

Again, since (Math. Monthly, Vol. II. p. 150)

$$\frac{\cos\delta'}{\cos\varrho'} = \cos r \left(\frac{-\sin a + \sin b + \sin c}{2\sin(s-a)}\right),$$

we have

$$\cot\tfrac{1}{2}A \frac{\cos\delta'}{\cos\varrho'} = \frac{\sin s(-\sin a + \sin b + \sin c)\cos r}{2n},$$

$$\cot\tfrac{1}{2}B \frac{\cos\delta''}{\cos\varrho''} = \frac{\sin s(\sin a - \sin b + \sin c)\cos r}{2n},$$

$$\cot\tfrac{1}{2}C \frac{\cos\delta'''}{\cos\varrho'''} = \frac{\sin s(\sin a + \sin b - \sin c)\cos r}{2n}.$$

$$(7) \qquad \therefore \cot\tfrac{1}{2}A \frac{\cos\delta'}{\cos\varrho'} + \cot\tfrac{1}{2}B \frac{\cos\delta''}{\cos\varrho''} + \cot\tfrac{1}{2}C \frac{\cos\delta'''}{\cos\varrho'''} = \frac{\sin s(\sin a + \sin b + \sin c)\cos r}{2n}.$$

But (CHAUVENET'S Trig. Eq. 332)

$$(8) \qquad \frac{\cos\delta}{\cos r \sin\varrho} = \frac{\sin a + \sin b + \sin c}{2n}, \quad \cos r \tan\varrho = \frac{n\cos r}{\sin s}.$$

Combining equations (8) we get

$$(9) \qquad \frac{\cos\delta}{\cos\varrho} = \frac{(\sin a + \sin b + \sin c)\cos r}{2\sin s}.$$

Combining (5) and (7)

$$(10) \qquad \cot\tfrac{1}{2}A \cot\tfrac{1}{2}B \cot\tfrac{1}{2}C \frac{\cos\delta}{\cos\varrho} = \frac{\sin s(\sin a + \sin b + \sin c)\cos r}{2n},$$

and combining (7) and (10) we have (3)

SOLUTION OF PART FIRST.

By GUSTAVUS FRANKENSTEIN, Springfield, Ohio.

The plane triangle $C = 180 - (A + B)$,

$$\therefore \cot \tfrac{1}{2} C = \cot\ (90 - \tfrac{1}{2}(A + B)) = \tan \tfrac{1}{2}(A + B)$$

$$= \frac{1}{\cot \frac{1}{2}(A + B)} = \frac{\cot \frac{1}{2} A + \cot \frac{1}{2} B}{\cot \frac{1}{2} A \cot \frac{1}{2} B - 1},$$

$$\therefore \cot \tfrac{1}{2} C (\cot \tfrac{1}{2} A \cot \tfrac{1}{2} B - 1) = \cot \tfrac{1}{2} A + \cot \tfrac{1}{2} B,$$

$$\therefore \cot \tfrac{1}{2} A \cot \tfrac{1}{2} B \cot \tfrac{1}{2} C = \cot \tfrac{1}{2} A + \cot \tfrac{1}{2} B + \cot \tfrac{1}{2} C.$$

SIMON NEWCOMB.
W. P. G. BARTLETT.
TRUMAN HENRY SAFFORD.

NOTES AND QUERIES.

1. *On some properties of the powers of the same number.* — The author announced the discovery of some general laws which regulate the series of the powers of any number. For instance, in the following series of the powers of 5, the number of digits in the several recurrent vertical series may be expressed by the powers of 2.

No. of digits recurring.	Powers of 5.		The vertical series are
	5	1st	5
1	25	2d	2
2	125	3d	16
	625	4th	3580
4	3125	5th	17956240
8	15625	6th	3978175584236200
	78125	7th	19840377976181556439582242163600
16 . . .	390625	8th	
32 . .	1953125	9th	
	9765625	10th	
64 . .	48828125	11th	
128 .	244140625	12th	
256 .	1220703125	13th	

The next consists of 64 figures, and so on. He pointed out that a similar law existed for every other number, and he exhibited formulæ by which the sum of any of the recurrent series may be de-

termined. In the case of 5, $S_n = 2(S_{n-1} + 1)$, the consecutive sums of the several series being 7, 16, 34, 70, 142, &c. In this way tables of the powers of numbers may be constructed to any extent whatever with very little labor. This discovery will enable certain calculations to be made with a degree of accuracy hitherto impossible. — J. Pope Hennessy, of the Inner Temple, Report of the British Assoc. for the Adv. of Science. Leeds. 1858.

2. The *celestial* part of the problem on page 204 of the March number of the Monthly is very simply solved in a general form. Employing the usual notation, S the integral with respect to the mass, Ω the *potential*, $\triangle$ the mutual distance of m and m', &c., *the law of power* of any system of *fixed* forces is

$$(1) \qquad \tfrac{1}{2} S(m v^2) = \Omega + H.$$

$$(2) \qquad \Omega = \mu S S[m m' \varphi(\triangle)]. \qquad (3) \quad \varphi(\infty) = 0.$$

Fixed forces might, it is true, be imagined which would not satisfy (2) and (3); but it can hardly be supposed that such would ever have been introduced into a system of nature.

The problem here considered is to find the form of the function φ, under the condition that two systems, whose linear dimensions are in the ratio n, shall present similar appearances at the same times. If (1) expresses the law of power of one system, that of the other is by this condition,

$$\tfrac{1}{2} S[n^3 m (n v)^2] = \mu S S[n^6 m m' \varphi(n \triangle)] + H_n.$$

Whence

$$S S m m' [n \varphi(n \triangle) - \varphi(\triangle)] = 0 \ldots$$

$$(4) \qquad \therefore n \varphi(n \triangle) - \varphi(\triangle) = 0.$$

The derivative of (4) with reference to n is

$$(5) \qquad \varphi(n \triangle) + n \triangle \varphi'(n \triangle) = 0.$$

The integral of (5) is $\varphi(n\triangle) = \frac{a}{n\triangle}$. The *only* form of potential which can satisfy the present problem is therefore that which results in *the law of gravity*. — B.

3. Reply to query on page 186, March No. For the integral of

$$D_r^2 \varphi(r) + \frac{D_r \varphi(r)}{r} \pm m^2 \varphi(r) = 0,$$

see De Morgan's *Calculus*, pp. 699, 700; Boole's *Differential Equations*, pp. 455, 459; or Carmichael's *Calculus of Operations*, p. 54. — B.

NOTE ON EQUATIONS OF THE SECOND DEGREE.

By Dr. R. C. Matthewson, U. S. Deputy Surveyor, San Francisco, Cal.

Every equation of the second degree, involving one unknown quantity, may be reduced to one of the following four forms, in which the coefficient of the second power of the unknown quantity will always be unity: —

1st, $x^2 - 2rx = a^2$; 2d, $x^2 + 2rx = a^2$;
3d, $x^2 - 2rx = -a^2$; and 4th, $x^2 + 2rx = -a^2$.

The roots of the 2d form differ from those of the 1st, and the roots of the 4th from those of the 3d, only in the change of signs; because if the signs of the alternate terms of an equation be changed, the signs of all the roots will be changed, while their magnitudes remain unchanged.

It is obvious, therefore, that the four forms may be reduced to two classes, the 1st class comprising the 1st and 2d forms, in which the absolute term is *positive*, and the 2d class comprising the 3d and 4th forms, in which the absolute term is *negative*.

In both classes, considered as lineal equations, one half of the coefficient of the first power of the unknown quantity represents

the *radius* of a circle, while the square root of the absolute term, in the 1st class, represents a *tangent*, and in the 2d class, a *sine* of that circle.

In the first class one of the roots of the equation represents the *external secant*, and the other the *diameter* of the circle *increased* by the *external secant*, while in the 2d class one of the roots represents the *versed sine* and the other the *diameter* of the circle *diminished* by the *versed sine*.

As there are no conditions in any of the four forms to indicate in what *direction* the lines are to be drawn, it follows that, in the 1st class, the external secant as well as the sum of the diameter and external secant, and in the 2d class, the versed sine as well as the difference of the diameter and versed sine, may be taken either positive or negative.

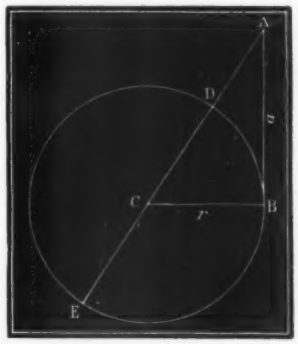
Hence, it is plain, that in each class, the two roots of one of the forms are respectively equal to the two roots of the other in magnitude, but have contrary signs, as has been already shown, and may be seen at once by comparing the roots together, as follows: —

Roots of 1*st Class.*

Form 1st, $x^2 - 2rx = a^2$, or $x = \quad r \pm \sqrt{r^2 + a^2}$,

Form 2d, $x^2 + 2rx = a^2$, or $x = -r \pm \sqrt{r^2 + a^2}$.

Construction of 1*st Class.*



With the radius $CB = r$, half the coefficient of x, describe a circle; draw the tangent $BA = a$, the square root of the absolute term; and through A and C draw the secant ACD produced, cutting the circumference of the circle in D and E; then AD and AE will be the roots of the equation, the former of which is the external secant and the latter the diameter of the circle increased by the external secant.

Demonstration of 1*st Class.*

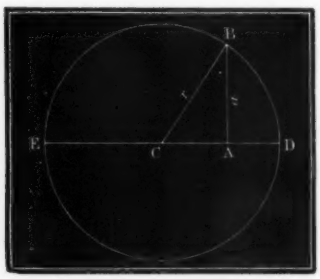
Form 1st, $AD = r - \sqrt{r^2 + a^2} =$ rad $-$ sec $=$ $-$ ext. secant,
" $AE = r + \sqrt{r^2 + a^2} =$ rad $+$ sec $=$ $+$ dia $+$ ext. sec.,
Form 2d, $AD = -r + \sqrt{r^2 + a^2} = -$ rad $+$ sec $=$ $+$ ext. secant,
" $AE = -r - \sqrt{r^2 + a^2} = -$ rad $-$ sec $=$ $-$ dia $-$ ext. sec.

Roots of 2*d Class.*

Form 3d, $x^2 - 2rx = -a^2$, or $x = r \pm \sqrt{r^2 - a^2}$,
Form 4th, $x^2 + 2rx = -a^2$, or $x = -r \pm \sqrt{r^2 - a^2}$.

Construction of 2*d Class.*

With the radius $CD = r$, half the coefficient of x, describe a circle; through the centre C draw a diameter cutting the circumference of the circle in D and E; draw the sine $AB = a$, the square root of the absolute term, and join B and C; then AD and AE will be the roots of the equation, the former of which is the versed sine and the latter the diameter of the circle diminished by the versed sine.



Demonstration of 2*d Class.*

Form 3d, $AD = r - \sqrt{r^2 - a^2} =$ rad $-$ cos $=$ $+$ versed sine,
" $AE = r + \sqrt{r^2 - a^2} =$ rad $+$ cos $=$ $+$ dia $-$ ver. sine,
Form 4th, $AD = -r + \sqrt{r^2 - a^2} = -$ rad $+$ cos $=$ $-$ versed sine,
" $AE = -r - \sqrt{r^2 - a^2} = -$ rad $-$ cos $=$ $-$ dia $+$ ver. sine.

SOLUTION OF PROBLEMS IN PROBABILITIES.

By Dr. Joel E. Hendricks, Newville, Indiana.

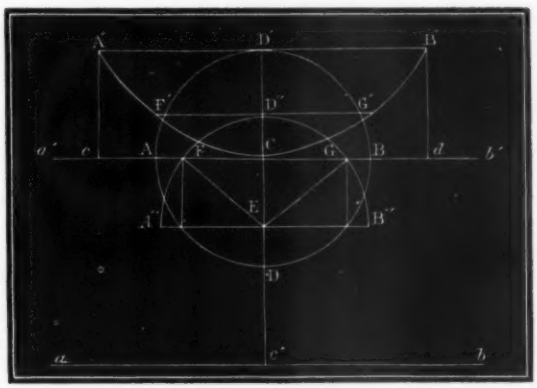
I have recently had my attention directed to a question somewhat analogous to the Prize Question proposed in the *Lady's and Gentle-*

man's Diary, for 1859, and published in the Mathematical Monthly, Vol. I. No. IV.; and have succeeded, as I think, in obtaining a solution, which, by a slight and obvious extension, will also apply to the "Prize Question." I submit the solutions for publication.

Prob. I. "A plane surface is ruled with parallel or equidistant lines; a slender rod, the length of which equals the perpendicular distance between two consecutive lines, is thrown at hazard upon the plane; determine the probability of its falling across a line."

Prob. II. "Two rods, two and four feet long respectively, having their middle points connected by a string one foot in length, are thrown up; show that their chance of crossing is $\frac{1}{2} + \frac{2}{\pi^2}$."

Solution. — Let $a\,b$, $a'\,b'$, represent two consecutive lines, and $C\,c'$ the perpendicular distance between them; also, let $C\,D = \frac{1}{2}\,C\,c'$ = one half the length of the rod.



The centre of the rod may fall anywhere on the line $C\,c'$, but as the position of the rod in relation to the line $a\,b$ will be the same if it fall below D, as to the line $a'\,b'$ if it fall above, we need only consider its position when the centre falls between C and D.

Let E be any point between C and D, and upon E as a centre and radius = $C\,D$ describe the semicircle $A''\,D''\,B''$, cutting the line $a'\,b'$ in F and G. Then it is obvious, that, in all positions between F and G, the rod will cross the line, and in all positions between F and A'', and G and B'', it will miss. Therefore, the probability of the rod's falling across the line $a'\,b'$ will be represented by a fraction

whose numerator is the sum of all the arcs above the line $a'b'$ made by causing E to assume all possible positions between C and D; and whose denominator is the sum of all the semicircles $A''D''B''$, &c., made by causing E to assume all possible positions between C and D. But the sum of all the arcs above $a'b'$ is represented by the area of the sinusoid $A'F'C G'B'$; and the sum of all the semicircles $A''D''B''$, &c., is represented by the area of the rectangle $A'B'cd$, which, when the radius $CD = 1$, is π.

Let the ordinate $F'D''$ (= arc FD') $= y$; then is $CD'' = 1 - \cos y$, and the area of the sinusoid $A'F'C G'B'$ is

$$2\int_0^{\frac{1}{2}\pi} y \sin y \, dy = \left[-2y\cos y + 2\sin y\right]_0^{\frac{1}{2}\pi} = 2.$$

Therefore the probability of the rods falling across a line is $\frac{2}{\pi}$.

In extending this solution to Problem II. we observe, (if $a'b'$ represent the long rod, $A''B''$ the short one, and CD the string,) that, supposing the long rod to fall in the position $a'b'$, the centre of the short rod may fall anywhere in the semicircle ADB. Therefore the chance of their crossing will be represented by a fraction whose numerator is the sum of all the products $F'G' \times FG$ obtained by making E assume all possible positions between C and D; and whose denominator is the sum of all the semicircles $A''D''B''$, &c., obtained by making E assume all possible positions in the semicircle ADB. Therefore the chance of crossing is

$$\frac{4}{\frac{1}{2}\pi^2}\int_0^{\frac{1}{2}\pi} y \sin^2 y \, dy = \frac{4}{\frac{1}{2}\pi^2}\int_0^{\frac{1}{2}\pi} y\left(\tfrac{1}{2} - \tfrac{1}{2}\cos 2y\right) dy$$

$$= \left[\frac{y^2 - y\sin 2y - \frac{1}{2}\cos 2y}{\frac{1}{2}\pi^2}\right]_0^{\frac{1}{2}\pi}$$

$$= \frac{\frac{1}{4}\pi^2 - 0 + \frac{1}{2} + \frac{1}{2}}{\frac{1}{2}\pi^2} = \frac{1}{2} + \frac{2}{\pi^2}.$$

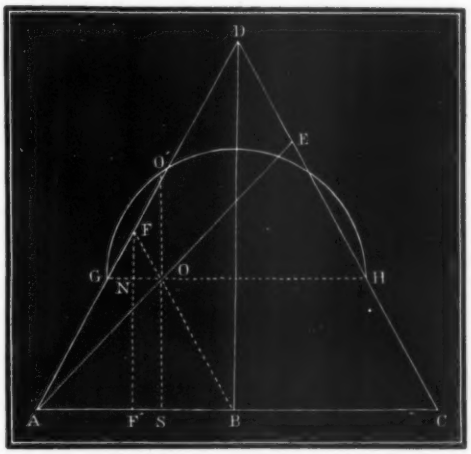
ON THE INTERPRETATION OF IMAGINARY ROOTS IN QUESTIONS OF MAXIMA AND MINIMA.

By Major Benjamin Alvord, U. S. Army.

We shall take the following example: —

To cut from a given right cone an ellipse whose surface is a maximum, or a minimum, by a plane passing through a fixed point on one of its elements.

Let D be the vertex, $A\,C$ the diameter of the base of the given right cone, and A the given fixed point. Suppose $A\,E$ to be the transverse axis of the required ellipse. Put $A\,B$, the radius of the base $= a$, and the altitude $B\,D = c$. Through B draw $B\,F$ parallel to $C\,D$. The centre, O, of the required ellipse must be found somewhere on $B\,F$. Through O draw a plane, $G\,H$, parallel to the base; it will cut out of the cone a circle, as $H\,O'\,G$, having $H\,G$ as a diameter. Erect the perpendicular $O\,O'$, which is the semi-conjugate axis of the required ellipse. The surface of the ellipse is measured by $\pi\,A\,O \times O\,O'$; or the surface is a maximum (or minimum) when the product $A\,O \times O\,O'$ is a maximum; or the square $A\,O^2 \times O\,O'^2$; or when $A\,O^2 \times O\,G \times O\,H$ is a maximum, since $O\,O'^2 = O\,G \times O\,H$. But $O\,H = B\,C = a$ is a constant. Thus the function is then expressed $u = A\,O^2 \times O\,G$. First, we have



$$(1) \qquad O\,G : A\,B = N\,F : F\,F'.$$

But $N\,F = \frac{c}{2} - O\,S$, $F\,F' = \frac{c}{2}$; and putting $O\,G = y$ (1) becomes

$$y : a = \frac{c}{2} - OS : \frac{c}{2}.$$

$$\therefore \frac{cy}{2} = \frac{ac}{2} - a \times OS;$$

$$\therefore OS = \frac{ac - cy}{2a} = \frac{c}{2} - \frac{cy}{2a}.$$

But $AS = AF' + F'S = \frac{a}{2} + \frac{y}{2}$;

$$\therefore AO^2 = OS^2 + AS^2 = \frac{c^2}{4} - \frac{c^2 y}{2a} + \frac{c^2 y^2}{4a^2} + \frac{a^2}{4} + \frac{ay}{2} + \frac{y^2}{4}.$$

$$\therefore u = AO^2 \times OG = \frac{c^2 y}{4} - \frac{c^2 y^2}{2a} + \frac{c^2 y^3}{4a^2} + \frac{a^2 y}{4} + \frac{ay^2}{2} + \frac{y^3}{4}.$$

$$\frac{du}{dx} = \frac{c^2}{4} - \frac{c^2 y}{a} + \frac{3c^2 y^2}{4a^2} + \frac{a^2}{4} + ay + \frac{3y^2}{4} = 0.$$

$$\therefore (3c^2 + 3a^2) y^2 - (4ac^2 - 4a^3) y = -a^4 - a^2 c^2;$$

$$(2) \qquad \therefore y = \frac{2a(c^2 - a^2)}{3(a^2 + c^2)} \pm \frac{a\sqrt{c^4 + a^4 - 14a^2 c^2}}{3(a^2 + c^2)}.$$

Finding the second derivative, we have

$$\frac{d^2 u}{dy^2} = \frac{3}{2}\left(\frac{a^2 + c^2}{a^2}\right) y - \frac{c^2 - a^2}{a}.$$

Substituting the value of y in (2) we get

$$\frac{d^2 u}{dy^2} = \pm \frac{\sqrt{c^4 + a^4 - 14a^2 c^2}}{2a}.$$

Hence, when the value of y is taken with the *plus* sign of the radical, it corresponds to the required minimum; but when the *minus* sign is taken, it corresponds to a maximum.

But the authorities lay it down (as in CHURCH's Calculus, Art. 67), that only the *real* roots of the equation $\frac{du}{dy} = 0$ answer to a maximum or a minimum. In order that the value of y, or of the second derivative, shall not become imaginary, it is necessary that $a^4 + c^4 - 14a^2 c^2$ should be positive. But this expression can be put under the form

$$(a^2 + c^2)^2 - 16\,a^2\,c^2 = (a^2 + c^2 + 4\,a\,c)\,(a^2 + c^2 - 4\,a\,c).$$

It is necessary then that $a^2 + c^2 > 4\,a\,c$. The limit of the relation between a and c will be found by putting $a^2 + c^2 = 4\,a\,c$.

(3) $\therefore\ c^2 - 4\,a\,c = -a^2$; or $c = 2\,a + a\sqrt{3} = a\,(3.732)$.

But the angle $A\,D\,B$, answering to these relations between the radius of the base and the altitude, is 15° 0′ 1″. Thus, in order that the problem may be possible, or that a maximum or minimum may exist, the angle $A\,D\,C$, or the angle between the two elements of the cone cut by a section through the axis, must be less than 30° 0′ 2″; or the altitude of the cone must be more than 3.732 times the radius of the base. In the first figure the angle $A\,D\,C$ is 57°, and hence in that example there is no maximum or minimum.

Take the example in the annexed figure. The angle $A\,D\,C$, at the vertex is 26°. It will be found that the position of the section for a maximum is $A\,K$, and for a minimum it is $A\,I$. If, in equation (3), the minus sign of the radical were taken, we should find that $c = a\,(0.268)$. In this case, the expression under the radical would not be imaginary if the altitude is less than 0.268 of the base; or, which is the same thing, if the base is more than 3.732 times the altitude; or the angle $A\,D\,C$ more than 149° 59′ 58″. In this case, $c - a$ in equation (2) being negative, the value of y is negative for both signs of the radical. Therefore y, or $G\,O$, in the first figure is laid off in the opposite direction, and the line $A\,E$ passes to the outside of the angle $C\,A\,D$. The problem in this case would have no reference to the cone, but could be announced in the following form. Having given an isosceles triangle, $A\,D\,C$, and $B\,F$ being drawn parallel to $C\,D$, required to draw a line, as $A\,E$, through the point A, such that the product $A\,O \times O\,G$ is a maximum or a minimum.

This discussion is given as a remarkable example in illustration of the fact, that if, in the investigation of questions in maxima and minima, an imaginary expression for the unknown quantity is found, it indicates that certain relations between the constants in the original function must be fulfilled in order that the problem may be possible.

LETTER OF LE VERRIER AND REMARKS OF FAYE UPON THE INTER-MERCURIAL PLANETS.

Translated by CHARLES HENRY DAVIS, Commander U. S. N., Supt. Nautical Almanac.

MY DEAR SIR:—

I have the pleasure to communicate to you a translation of the remarkable letter of M. LEVERRIER, published in the *Comptes Rendus*, No. 11, Deuxième Semestre, 1859, together with M. FAYE'S remarks upon this occasion.

The important bearing of these papers upon the total eclipse of next July, induces me to ask you to bring them, in the pages of your Journal, to the notice of astronomical observers throughout the country.

I am, very respectfully,

Your friend and obedient servant,

CHARLES HENRY DAVIS,
Commander U. S N., Superintendent N. A.

To J. D. RUNKLE, A. M., A. A. S.,
Editor of the Mathematical Monthly, Assistant N. A.

LETTER OF M. LEVERRIER TO M. FAYE UPON THE THEORY OF MERCURY, AND UPON THE MOTION OF THE PERIHELION OF THAT PLANET.

You have probably not forgotten, that, in my investigations into the motions of our planetary system, I have encountered difficulties in producing a complete agreement between theory and observa-

tion. This agreement, said BESSEL, thirty years ago, has always been asserted, without, however, having ever been confirmed in a manner sufficiently substantial.

The known errors in the motion of Uranus have been explained by the discovery of Neptune.

The investigation of the difficulties presented by the Sun has been long and intricate. It was necessary first to revise the catalogue of the fundamental stars, in order to exclude from it any systematic error. I have resumed, subsequently, the whole theory of the inequalities of the Earth; after which, I have been led to the consecutive discussion of as many as nine thousand observations of the Sun, made in different observatories. This work has proved, that the meridian observations have not perhaps always possessed the accuracy attributed to them; and thus the errors, described at first as belonging to theory, must in the end be thrown back on the uncertainty of the observations.

The theory of the Sun being put out of the question, the study of the motions of Mercury could be taken up with advantage. And it is concerning this undertaking that I wish to address you at present.

While we possess, in respect to the Sun, meridian observations only, liable to great objections, we arrange in order, through the space of a century and a half, a certain number of observations of Mercury, possessing great precision. I am speaking here of the internal contacts of the disc of Mercury with the disc of the Sun, when the planet passes in front of the latter. Provided the place of observation has been well known, provided the astronomer has been furnished with a decent telescope, and his clock has been regulated within some seconds, the knowledge of the moment when the internal contact took place ought to furnish the means of estimating the distance between the centres of the planet and of the

Sun, without a greater error than one second of arc. We have, from 1697 to 184Ɛ, twenty-one observations of this sort, which we ought to be able to satisfy in the strictest manner, if the inequalities in the motions of the Earth and of Mercury have been correctly computed, and if the values assigned to the disturbing masses are exact.

In my first researches on Mercury, published in 1842, the observations of the transits were not produced with such great accuracy. There might be observed, among other errors, in relation to the transits of the month of May, a somewhat remarkable progressive error, which amounted to 9 seconds of arc in 1793. Such discrepancies could not be attributed to errors of observation. But, not having then revised my theory of the Sun, I thought it best to refrain from drawing any conclusion from this fact.

The use of the corrected tables of the Sun, in my new work, has not caused the errors above described to disappear entirely; systematic errors, which could not be ascribed to the observations without supposing that astronomers, such as Lalande, Cassini, Bouguer, &c., had committed errors of several minutes of time, having even a progressive variation from one epoch to another, — an impossibility!

But it is worthy of remark, that an addition of thirty-eight seconds to the secular motion of the perihelion is sufficient to represent all the observations of the transits within less than a second, and even the greater part within less than half of a second. This neat result, which immediately gives to all the comparisons a precision superior to that hitherto arrived at in astronomical theories, shows clearly that this increase of the motion of the perihelion of Mercury is indispensable, and that with it the tables of Mercury and the Sun possess all the accuracy required.

The necessity for adding thirty-eight seconds to the secular mo-

tion of the perihelion of Mercury being once admitted, let us inquire to what conclusions it will lead us. As the motion originally adopted for the perihelion proceeded from received values of the masses of the disturbing planets, we ought first to inquire what changes it would be necessary to apply to these masses in order to increase the computed motion by thirty-eight seconds. Now we perceive that this would only be possible upon one condition, namely, to increasé the mass assigned to Venus *by at least a tenth* of its value. Is this alteration admissible?

If we derive the mass of Venus from the periodic perturbations which that planet occasions to the Earth's motion, we find, from the discussion of numerous meridian observations of the Sun, made between 1750 and 1810, that this mass is the *four-hundred-thousandth* part of that of the Sun. We reach the same result by taking into account the observations made between 1810 and 1850. It is the mass which we have adopted, and which it would be necessary to increase by *a tenth*, according to the discussion of the transits of Mercury over the Sun.

The perturbative action of Venus is again perceptible in the secular variation of the obliquity of the ecliptic; and if we deduce this variation from the seven solstices observed with the greatest accuracy, from the time of Bradley to the present day, we find that the mass of Venus, which we have just quoted, is *a little too great, — a result contrary to that given by Mercury.* This contradiction is the point to which we are to give our attention.

If we inquire how the seven solstices, observed since Bradley, could be represented, supposing a variation of the obliquity of the ecliptic which would correspond to a mass of Venus increased by *one tenth*, we perceive that it is impossible to avoid errors of *two seconds and a half* in the measured value of the obliquity. It seems difficult to admit this result, the more especially that the errors

would vary progressively from BRADLEY to our own epoch, which would in fact establish a difference of five seconds between the extreme observations.

A grave difficulty then results from the comparison of the theories of the Earth and of Mercury, which appear to involve different values for the mass of Venus. If we accept the mass given by the observations of Mercury, we must conclude, either that the secular variation of the obliquity of the ecliptic, deduced from observation, contains errors that are very improbable, or else that this obliquity undergoes changes from other causes, not yet recognized by us. If, on the other hand, we consider the variation of the obliquity and the causes by which it is produced as perfectly settled, we shall be induced to think that the excess in the motion of the perihelion of Mercury is owing to some agency still unknown, "cui theoriæ lumen nondum accesserit."

It is far from my intention to decide absolutely between these hypotheses. It has been my wish only to show that there is a serious difficulty here, that deserves to arrest the attention of astronomers, to become the subject of their deep thought, and to furnish matter for a weighty discussion. In order to take the first step in this direction, I will observe that it is not apparent what disturbing cause could derange the obliquity of the ecliptic, without, at the same time, exerting very conspicuous effects upon the secular variations of the elements of motion of the planets, — effects which have not been noticed, — whilst it would be possible to conceive a cause capable of producing in the perihelion of Mercury the thirty-eight seconds of secular motion required, and which would produce in the planetary system no other sensible effect whatever.

To fix our thoughts, let us consider a planet which would be situated between Mercury and the Sun, and, as we have not observed a variation in the motion of the node of the orbit of Mercury

similar to that of the perihelion, let us conceive that the supposed planet moves in an orbit but little inclined to the orbit of Mercury. Let us even conceive, on account of the indeterminateness of the problem, that the orbit is circular.

As the hypothetical planet must produce a secular motion of thirty-eight seconds in the perihelion of Mercury, it follows that there is a relation between its mass and its distance from the Sun, such that, in proportion as we assume the distance to be smaller, the mass will increase, and inversely. For a distance a little less than half of the mean distance of Mercury from the Sun, the mass sought for will be equal to that of Mercury.

But is it possible such a body could exist without having ever been noticed? Assuredly it would be endowed with great brilliancy. Is it to be believed that in consequence of its slight elongation it would be always lost in the diffused light of the Sun? How conceive that we have never been struck with its brilliant light during some one of the total eclipses of the Sun? How does it happen that we have never discovered it passing over the disc of the Sun?

All these difficulties vanish upon the admission, in the place of a single planet, of the existence of a series of small bodies (corpuscules) circulating between Mercury and the Sun.

Under the *mechanical* aspect, the influence of all these small bodies would be united to produce the required motion of the perihelion of Mercury; and, supposing always that they moved in circles, they would exert no effect upon the eccentricity of the orbit of this planet. As they would be distributed over all parts of the ring that they would form, the periodic influences which each one would exert upon Mercury would mutually destroy each other.

Under the *physical* aspect, it would not be astonishing if the regions which border on the Sun should be found to be less free than

the remainder of the planetary system. Since there circulates between Jupiter and Mars a ring of small bodies, of which the largest alone have been seen in our telescopes; since everything leads us to the belief that the vicinity of the orbit of the Earth is furrowed by innumerable groups of asteroids, it is altogether natural to suppose that the same formation may be reproduced within the orbit of Mercury. Could any of these bodies be sufficiently noticeable to be perceived in their transits over the disc of the Sun? Astronomers, already so observant of all the phenomena which exhibit themselves upon the surface of this star, will undoubtedly find, in these reflections, an additional motive for following with attention the smallest and best defined spots. Some minutes of observation will be usefully employed in deducing their nature from observing their motions.

Here, then, my dear associate, is a new complication which shows itself in the neighborhood of the Sun; there, where M. Encke has already pointed out so important a one to us in the subject of his comet of short period. This induces me to hope that both you and he will bestow some attention upon my conclusions, and direct upon them the light of discussion.

Remarks of M. Faye on the Occasion of M. LeVerrier's Letter.

The unexpected result of these profound researches, resumed for the second time with new elements, cannot fail to make a lively impression upon astronomers, and to promote the new examinations which M. LeVerrier himself suggests with an urgency, the motives for which are so fully explained. As one of the hypotheses on which the learned author appears to rest, to account for the motion of the perihelion of Mercury, points to an almost immediate verification, in which observers will first interest themselves, I shall ask the permission of the Academy to submit from this time a sort

of plan of operation. I refer here to the probable existence of a series of small planets within the orbit of Mercury.

A new planet has often been sought for in these dazzling regions, but at a venture, and always in vain. The failure proves nothing, because these researches were merely fanciful. Under the impulse of a weighty probability, the result might be altogether different, provided the operation was conducted upon a plan rationally conceived. And, in the first place, it is evident that the brilliancy of the heavens in the circumsolar region would only have allowed the discovery in this manner of a star of the order of Mercury itself, and not the small planets indicated by M. LeVerrier. We are, then, first led to turn to account the darkness of total eclipses, and particularly that of July next, which is about to afford us an opportunity of trying a first experiment. We know, it is true, that during the greater part of these eclipses we see with the naked eye scarcely anything but the planets and the most brilliant stars. But this fact is explained in a great measure by the duration of the glare. If the observer, instead of following the Sun up to the last moment, remained in darkness a quarter of an hour before the total eclipse, his eye would be much more sensitive at the decisive moment. Let us suppose that an astronomer takes charge of this research at one of the stations of Spain * or of Algeria, where we shall go next year to observe this magnificent phenomenon; let us suppose, moreover, that he is furnished with a good comet-seeker mounted as an equatorial or a theodolite, so as to determine at necessity a direction with a certain degree of accuracy; let us suppose, finally, that he gives up the pleasure of observing the most curious phases, and that he keeps for some time in nearly complete

* The station of Campvey, already celebrated through the geodetic labors of MM. Biot and Arago, would offer some advantages in this particular.

darkness: he will be entirely ready to catch the least sparkle in the circumsolar region, beyond the aureola; and the few minutes' duration of the total eclipse would be sufficient for him to explore a large part of the region designated by M. LeVerrier.

Total eclipses are sufficiently frequent to prevent a successful observation from remaining a long time isolated; if, then, the result of the next eclipse is not negative, I do not doubt but that we shall very soon be able to obtain some accurate notions concerning the bodies that will be discovered under these particular circumstances.

Be it as it may with regard to this expectation, I have thought it my duty to insist upon a condition of success which has always been neglected, that of avoiding the duration of the dazzle caused by the direct observation of the Sun, or even by the simple effect of daylight.

The learned Director of the Observatory himself points out another method quite as effectual, perhaps. It consists in following with care the small spots with which the Sun is frequently strewed. Planets, of which the orbit would be very little inclined to that of Mercury would have, in fact, like the latter, their transits across the Sun; transits which for a long time might escape the attention of astronomers not forewarned, as did a certain satellite of Saturn in a region as limited and not less explored. But the difficulty even of these researches induces me to return to a suggestion of Sir John Herschel which I have very often called to mind. If, in several observatories suitably selected, pains were taken to photograph the Sun many times a day, with the aid of a large instrument, we should obtain almost a continuous history of the disc of this body, and no one of the phenomena to which M. LeVerrier has just alluded could escape the observer. I have myself shown how the value of an astronomical observation might be given to

these photographs, independently of all apparatus of measure, by taking two impressions upon the same plate, at an interval of two minutes. The beautiful trials of the eclipse of the 15th of March have given beforehand an assurance of success. It will be sufficient to superimpose transparent negatives of this size, taken at intervals of a quarter of an hour, to distinguish immediately the movable projection of an asteroid in the midst of the most complicated groups of small spots. This undertaking would not in any manner debar the usual investigations upon the solar disc, and would retain, even if it should fail for the inter-Mercurial planets, an immense value for the physical history of the Sun itself.

APPLICATION OF THE BINOMIAL THEOREM TO THE EXTRACTION OF THE ROOTS OF WHOLE NUMBERS.

By Theodore Strong, Professor of Mathematics and Natural Philosophy in Rutgers College, New Brunswick, N. J.

If a, b, c, &c. stand for the successive digits of the number, and 10^m represents the local value of a; then the number will be represented by the forms

$$(1)\ a10^m+b10^{m-1}+c10^{m-2}+\&c.=a10^m\left(1+\frac{b10^{-1}+c10^{-2}+\&c.}{a}\right)=N,$$

noticing that instead of (1) it will sometimes be important, especially when a is greater than 5, to write

$$10^{m+1}-[10^{m+1}-(a\,10^m+b\,10^{m-1}+\&c.)]=N,$$

which is easily changed to the form

$$(2)\qquad 10^{m+1}[1-(a'\,10^{-1}+b'\,10^{-2}+\&c.)]=N.$$

Since

$$1+\frac{b\,10^{-1}+c\,10^{-2}+\&c.}{a}\quad\text{and}\quad 1-\frac{a'\,10^{-1}+b'\,10^{-2}+\&c.}{1}$$

are of the forms

$$1 + \frac{B}{A}, \text{ and } 1 - \frac{C}{D},$$

they may be reduced to products of the forms

$$\left(1 + \frac{B'}{A'}\right)\left(1 + \frac{B''}{A''}\right) \text{ &c.}, \quad \left(1 - \frac{C'}{D'}\right)\left(1 - \frac{C''}{D''}\right) \text{ &c.},$$

such that

$$\frac{B'}{A'}, \frac{B''}{A''}, \text{ &c.}, \quad \frac{C'}{D'}, \frac{C''}{D''}, \text{ &c.},$$

may be of any required degree of smallness.*

Again, since

$$2 = 1 \div \tfrac{1}{2} = 1 \div (1 - \tfrac{1}{2}) = (1 - \tfrac{1}{2})^{-1},$$

we have

$$3 = (1 - \tfrac{1}{2})^{-2}(1 - \tfrac{1}{4}), \qquad 4 = (1 - \tfrac{1}{2})^{-2},$$
$$5 = (1 - \tfrac{1}{2})^{-2}(1 + \tfrac{1}{4}), \qquad 6 = (1 - \tfrac{1}{2})^{-3}(1 - \tfrac{1}{4}),$$
$$7 = (1 - \tfrac{1}{2})^{-3}(1 - \tfrac{1}{8}), \qquad 8 = (1 - \tfrac{1}{2})^{-3},$$
$$9 = (1 - \tfrac{1}{2})^{-3}(1 + \tfrac{1}{8}), \qquad 10 = (1 - \tfrac{1}{2})^{-3}(1 + \tfrac{1}{4}), \text{ &c.}$$

These equations show that the digit a can be reduced to binomial forms, such that the second terms of the binomials shall not numerically exceed the fraction $\frac{1}{2}$; and that for 10^m we may write $(1 - \frac{1}{2})^{-3m}(1 + \frac{1}{4})^m$. From the binomial theorem we readily get the forms

$$(1 + x)^{\frac{r}{n}} = 1 + \frac{r}{n}x + \frac{\frac{r}{n}\left(\frac{r}{n} - 1\right)}{1 \cdot 2}x^2 + \frac{\frac{r}{n}\left(\frac{r}{n} - 1\right)\left(\frac{r}{n} - 2\right)}{1 \cdot 2 \cdot 3}x^3 + \text{ &c.}$$

$$(3) \qquad = 1 + \frac{r}{n}x + \frac{r(r-n)}{1 \cdot 2\, n^2}x^2 + \frac{r(r-n)(r-2n)}{1 \cdot 2 \cdot 3\, n^3}x^3 + \text{ &c.},$$

in which x and r may be positive or negative.

EXAMPLES.

1. "To extract the 100th root of 100."

Because $100 = 10^2$ we have

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* See Dr. STRONG'S Algebra, page 288. — ED.

$$\sqrt[100]{100} = 10^{0.02} = (1-\tfrac{1}{2})^{-0.06}\,(1+\tfrac{1}{4})^{0.02} = (1-0.5)^{-0.06}(1+0.25)^{0.02};$$

consequently, putting -0.5 for x and -0.06 for $\frac{r}{n}$, the first form of (3) gives

$$\begin{aligned}(1-0.5)^{-0.06} = 1 &+ 0.03 + 0.00795 + 0.0027295 + 0.001044033 + 0.000423877\\ &+ 0.000178735 + 0.000077364 + 0.000034136 + 0.000015285\\ &+ 0.000006924 + 0.000003166 + 0.000001458 + 0.000000676\\ &+ 0.000000315 + 0.000000154 + 0.0000000724 + \&c. = 1.042465746,\end{aligned}$$

the first seven decimals of which are correct.

Similarly, putting 0.25 for x and 0.02 for $\frac{r}{n}$, the first form of (3) gives

$$\begin{aligned}(1+0.25)^{0.02} = 1 &+ 0.005 - 0.0006125 + 0.000101062 - 0.000018823 + 0.000003746\\ &- 0.000000777 + 0.000000165 - 0.000000036 + \&c. = 1.004472837,\end{aligned}$$

the first seven decimals being correct.

Hence,

$$\sqrt[100]{100} = 1.04246574 \times 1.004472837 = 1.04712852,$$

the first seven decimal places of which are correct.

2. "To extract the fifth root of 281950621875."

Since the number equals

$$10^{11} \times 2.81950\ \&c. = 10^{10} \times 4 \times 5 \times 1.4097\ \&c.,$$

we immediately get

$$\begin{aligned}\sqrt[5]{28195\ \&c.} &= 10^2\,(1-\tfrac{1}{2})^{-\frac{1}{5}}\,(1-\tfrac{1}{2})^{-\frac{1}{5}}\,(1-\tfrac{1}{2})^{-\frac{1}{5}}\,(1-\tfrac{1}{2})^{\frac{1}{5}}\,(1+\tfrac{1}{4})^{\frac{1}{5}}\,(1+0.4097\ \&c.)^{\frac{1}{5}}\\ &= 200\,(1+\tfrac{1}{4})^{\frac{1}{5}}\,(1-0.29513)^{\frac{1}{5}} = 200\,(1-0.11891\ \&c.)^{\frac{1}{5}}\\ &= 200\,(1-0.11891\ \&c.)^{0.2}.\end{aligned}$$

Putting -0.11891 &c. for x, and 0.2 for $\frac{r}{n}$ in the first form of (3), we get

$$(1-0.11891\ \&c.)^{0.2} = 1 - 0.02378\ \&c. - 0.00056\ \&c. - \&c.;$$

consequently, we shall have

$$\sqrt[5]{28195}\ \&c. = 200\ (1 - 0.02378\ \&c. - 0.00056\ \&c. - \&c.).$$

Because the preceding series, as the number of terms is increased, gives 195 as the limiting value of $\sqrt[5]{28195}$ &c., we infer that 195 is the exact root; indeed, since the excess of nines in the given number and in the product $195 \times 195 \times 195 \times 195 \times 195$ are equal to zero, 195 must clearly be the exact root.

3. "To extract the seventh root of 2, correct to four decimal places."

Because $1 - \frac{1}{2}$ is of the form $1 - \frac{C}{D}$, it is easily resolved into the product $(1 - \frac{1}{4})\,(1 - \frac{1}{8})$; consequently, we shall have

$$\sqrt[7]{2} = (1 - \tfrac{1}{8})^{-\frac{1}{7}}\,(1 - \tfrac{1}{4})^{-\frac{1}{7}}.$$

Putting $-\frac{1}{8}$ for x, and -1 and 7 for r and n in the second form of (3), we get

$$(1-\tfrac{1}{8})^{-\frac{1}{7}} = 1 + 0.047619 + 0.009070 + 0.002159 + 0.000565 + 0.000156 + 0.000044 + \&c. = 1.059603;$$

and in like manner

$$(1 - \tfrac{1}{4})^{-\frac{1}{7}} = 1 + 0.035714 + 0.005102 + 0.000911 + 0.000160 + 0.000038 + \&c. = 1.041945.$$

Hence, we shall have

$$\sqrt[7]{2} = 1.059603 \times 1.041945 = 1.104048 +,$$

the first four decimals being correct.

Remark. — We have resolved $1 - \frac{1}{2}$ into factors for the purpose of obtaining series of more rapid divergency than the development of $(1 - \frac{1}{2})^{-\frac{1}{7}}$ will give.

Mathematical Monthly Notices.

Logarithmic Tables of Numbers and Trigonometrical Functions. By Baron von Vega. Translated from the Forty-third, or Dr. Bremiker's thoroughly revised and enlarged edition, by W. L. F. Fischer. Stereotyped. One volume octavo. Berlin: Weidmanns. 1859. New York: B. Westermann & Co. Price, $ 2.25.

Dr Bremiker's edition of Vega's Logarithmic and Trigonometrical Tables of seven decimal figures is unquestionably the best work of the kind that has yet been published. It is a "table portative," of the same order (and about the same size) as that of Callet, — so long popular on account of its superiority over preceding works, — but far surpasses that work in every requisite of a convenient table. 1st. In the clearness of the type. 2d. In the arrangement and grouping of the logarithms on the page, so as to cause the least fatigue to the eye in tracing a line across the page. 3d. In the facility of interpolating afforded by the tables of proportional parts in the trigonometrical portion, which are not given at all in Callet. 4th. In the accuracy with which the seventh place of decimals is determined in consequence of the preservation of the eighth place in the tables of proportional parts. 5th. In the improved arrangement of the table of log. sines, and tangents to every second for the first five degrees of the quadrant. 6th. In the very simple and convenient, as well as extremely accurate, means of obtaining the log. sines, and tangents of small angles (those less than 2° 46′ 40″ or 10,000″), afforded by the numbers marked S and T in the margin of the tables of logarithms; and, lastly, in the accuracy with which the tabular logarithms themselves are given. In this last respect, the tables of Callet have possessed a high reputation, as they were stereotyped as long ago as 1795, and are presumed to have been corrected in the stereotype plates whenever an error has been discovered. But Dr. Bremiker has entered into a critical examination of the seventh decimal place, carefully comparing the logarithms with those of Vega's *Thesaurus Logarithmorum* of ten figures, and recomputing to fifteen places all those logarithms of numbers in the Thesaurus which terminated in 500, in order to decide whether the seventh figure should be increased by a unit; and, for the same purpose, recomputing to fifteen places all the logarithms in the trigonometrical portion which terminated in 496, 497, 498, 499, 500, 501, 502, 503, and 504. A number of errors were thus detected, which have run through all tables down to the present time.

It was a happy idea to publish the tables with an English preface and introduction, the tables themselves being printed from the same stereotype plates as the original German edition, thus making them available to English readers without incurring renewed risks of errors in reprinting, and at the same time keeping the price of the book within the means of everybody.

While the work is admirably suited to the wants of the practical computer, we would also commend it especially to the student who desires to possess at least one good logarithmic table.

The Lady's and Gentleman's Diary, or Poetical and Mathematical Almanac for the Year 1860. London: Printed for the Company of Stationers. Price 1s. 4d.

This is one of the most valuable numbers of "The Diary." All the solutions of the problems proposed in 1859 are excellent, and the new problems are unusually interesting. We are glad to see the solutions of the Prize Question for 1859, by Messrs. Simon Newcomb and G. B. Vose, in the number before us. We append the Prize Question for this year.

Prize Question. — "If an hypocycloid has the radius of its describing circle one third of that of its base, being thus composed of three branches, and a straight line equal to twice the

diameter of the small circle be placed with its extremeties on two of the branches, it will touch the third branch."

A Treatise on Elementary and Higher Algebra. By THEODORE STRONG, LL.D., Professor of Mathematics and Natural Philosophy in Rutgers College, New Brunswick, N. J., Member of the American Philosophical Society, and of the American Academy of Arts and Sciences. New York: Pratt, Oakley, & Co., No. 21 Murray St. 1859.

Our readers will not need to be informed that DR. STRONG has devoted a long life to the successful study of the Mathematics, and that his name will ever be honorably identified with the progress of the science in this country. In the volume before us we find many most valuable results of this study which must be considered as positive additions to the science; and we regret that our space prevents more than a simple reference to some of the more important ones.

The roots of an equation are developed into series, by which it is readily shown that every equation of the nth degree must have n roots. This difficult and important proposition is thus brought within the domain of simple algebra. The solutions of Binomial Equations and the irreducible case of Cubic Equations are effected by pure algebra. A new and general method is given for the determination of the roots of equations. These are among the more important additions; but we would also call attention to the use of Detached Coefficients; to the Appendix to Multiplication and Division; to the Fundamental Principle of Single and Double Position, and of the Differential Calculus with applications; to a new investigation of the Binomial Theorem; to a new method of solving Quadratics, &c.

This is no ordinary work, and to those who like rigorous demonstration and thorough investigation we heartily commend it. We have but one disparaging remark to make. The arrangement of the matter is bad; the equations, prominent as well as less important, are printed and numbered in the text, often causing trouble to find the reference. The ease with which the mathematics are read depends much more upon arrangement than might at first seem; for the logical steps may often, to a great extent, be exhibited to the eye in the arrangement.

The Surveyor's and Engineer's Companion; being a concise Treatise on Mathematical Instruments, containing an improved method of telescopic measurements, illustrated with numerous engravings; and including the most important and useful tables and formulæ constantly used in surveying and engineering. By WILLIAM SCHMOLZ, Mathematical-Instrument Maker, San Francisco.

This is a small pocket field-book of 62 pages. The first part is mainly devoted to the descriptions of mathematical and scientific instruments, with rules for their adjustment and use, a large proportion of instruments being illustrated with very good wood engravings. The second part, prepared by Dr. R. C. MATHEWSON, U. S. Deputy Surveyor, San Francisco, can best be described by giving his full title-page. "A new set of Practical Tables, useful in Surveying and Engineering, containing Easy and Accurate Methods for finding the Variation of the Magnetic Needle at any Hour of the Night, Latitudes and Longitudes of Places from their Differences of Latitude and Departure, the Convergencies of the Meridians, the Divergencies of the Parallels of Latitude and Prime Verticals, Altitudes by the Barometer, Atmospheric Refraction, &c., together with *an Improved Method of Tabling*, which facilitates the Computation of Areas and the Projection of Maps."

The small size of this volume, made so by a judicious selection of matter, strongly recommends it.

Annual Report of the Board of Regents of the Smithsonian Institution, showing the Operations, Expenditures, and Condition of the Institution, for the Year 1858. Washington: James B. Steedman, Printer. 1859.

This Report begins, as usual, with that of the Secretary, Prof. HENRY, giving an account of the doings of the Institution during the year, embracing, besides the statement of its scientific operations and publications, an account of its Magnetic Observatory, Laboratory, System of Exchanges, Library, Museum, Gallery of Art, and Lectures. We are glad to learn that a catalogue of the series of Transactions and Proceedings of Learned Bodies now in the library of the Institution, prepared under the direction of the late Prof. W. W. TURNER, the learned and accomplished librarian of the Patent Office Library, is soon to be distributed. In the Appendix, "the object of which is to illustrate the operations of the Institution by the reports of lectures and extracts from correspondence, as well as to furnish information of a character suited especially to the meteorological observers and other persons interested in the promotion of knowledge," we find four lectures on astronomy; a memoir of Priestley; instruction for collecting insects; grasshoppers and locusts of America; means of destroying the grasshopper; vegetable colonization of the British Isles; causes which limit vegetable species towards the north, in Europe, and similar regions; forests and trees of North America; list of birds of Nova Scotia; atmospheric electricity; recent progress in physics; meteorology, and correspondence. We wish especially to call attention to the admirable lectures on astronomy, by Prof. ALEXIS CASWELL. They are indeed model "popular lectures," and a more clear and accurate notion of "the figure and magnitude of the earth," "the law of gravitation," "the dimensions of the solar system, or the extent of our knowledge of planetary distances," and "sidereal astronomy," can hardly be desired. They might be made the basis of a valuable text-book on astronomy.

A Treatise on the Higher Plane Curves; intended as a sequel to a Treatise on Conic Sections. By Rev. GEORGE SALMON, M. A., Fellow and Tutor, Trinity College, Dublin. Dublin: Hodges and Smith, Grafton Street, Booksellers to the University. 1852.

Our attention has recently been recalled to the above volume by a request from a correspondent that we would recommend a good work on the subject of which it treats.

We cannot better express our high opinion of the work before us than by giving the remarks of JAMES HENTHORN TODD, D.D., President of the Royal Irish Academy, upon the occasion of the presentation of the Cunningham Medal to Mr. SALMON, May 24, 1858.

The President said: — "The chief merit of Mr. SALMON'S 'Treatise on the Higher Plane Curves' is the clear and full exposition of all the modern improvements in the methods of analytical research which it contains. The author does not profess to have made any new discoveries, or to suggest new methods of investigation, but he has done both; and this new matter is introduced with so little parade, or, I should rather say, generally without any notice at all, that it requires considerable knowledge of the subject to distinguish the discoveries of ARONHOLD, or PLÜCKER, of PONCELET, or JOACHIMSTHAL, from the new and highly interesting propositions introduced whilst giving an account of the investigations of these authors, and connected with their research, but due altogether to Mr. SALMON.

"The method of investigating the properties of conic sections by reference to two tangents and the line joining their points of contact, and the analogous method applied to cubics of the third class; the application of the theory of determinates to the discovery of the properties of curves, particularly to finding the reciprocals of curves of the third and fourth degrees; the investigation of the focal properties of cubical biquadratic curves, and many other new properties of conics, may be mentioned as original, the result of Mr. SALMON'S genius and research. But, however valuable and interesting these additions to what previous writers had discovered, the great practical value of the work is, that it arranges in a clear and connected system all the important geometrical discoveries hitherto published, so that, to use the author's own words, 'each new student who wishes to devote himself to original investigation in any branch of

mathematics, may have his energies brought to bear upon the undiscovered parts of the science.' Such a student, with the aid of Mr. SALMON's book, without the labor of searching the scattered papers in scientific journals, or transactions of societies, will at once see what has been already done, and will escape the danger of wasting his abilities by rediscovering what others had discovered long before.

"It would be premature to speak of Mr. SALMON's investigations in the geometry of three dimensions, as they have not as yet been fully given to the public; but we have already had a foretaste of what may be expected from him in this higher region of mathematical research, as he has already read to the Academy a valuable paper on the 'Reciprocal of a Surface of the Second Degree,' and he has contributed to different periodical journals most important investigations relative to the surface of the third degree; all of which, with many additions, we may look forward to, in a collected form, in a third volume of his Geometry, devoted to the properties of surfaces."

Editorial Items.

THE following gentlemen have sent us solutions of the Prize Problems in the January number of the Monthly.

M. K. BOSWORTH, Marietta College, Ohio. Probs. III., IV., and V.

ASHER B. EVANS, Madison University, N. Y. Probs. I., II., III., IV., and V.

CADET H. S. WETMORE, U. S. M. Academy. Prob. I.

GUSTAVUS FRANKENSTEIN, Springfield, Ohio. Probs. III., IV., and part of V.

T. E. TOWER, Amherst College. Probs. III. and IV.

E. S. STEARNS, Chester Institute, N. J. Probs. I. and II.

W. H. SPENCER, C. J. BALDWIN, and H. A. WOOD, Madison University Grammar School. Each Prob. I.

T. S. HUBBARD, Alfred Centre, N. Y. Probs. I. and II.

W. F. OSBORNE, Wes. Univ., Middletown, Ct. Probs. III., IV., and V.

D. G. BINGHAM, Ellicotteville, N. Y. Probs. II. and III.

D. M. HUDSON, Paris, Ind. Probs. I., II., and III.

JAS. M. INGALLS, Delton, Sauk Co., Wis. Probs. I., II., and III.

JAMES F. ROBERSON, Ind. Univ. First part of Prob. V.

J. K. UPTON, New London Inst., N. H. Probs. I., II., and III.

We have not given in this number of the Monthly the Method of Integration by Quadratures, as we intended, in order to make room for the very interesting and important communication from COMMANDER DAVIS on the question of the inter-mercurial planets. We will simply add that, from the chart of the total eclipse of July 18, 1860, just issued from the Office of the Nautical Almanac, we learn that Astoria, Olympia, Fort Okonagon, and Fort Colville, on our northwest coast, Fort York on Hudson's Bay, and the northern point of Labrador, are within the limits of the total phase; and that the eclipse ought to be observed from one or more of these points, having in view, to settle if possible, the question discussed in the communication to which we have referred.

Erratum. — In the February No., last equation of Prize Problem V., in the denominators for $\cos\rho'$, &c., read $\sin\rho'$, &c.

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Notices.

A work which for nearly half a century has kept unflinchingly on the even tenor of its way, with a constantly growing reputation, has no need of culling "*notices*" from the Press. A single extract from a notice of the November number of 1859, from the Boston Courier and National Intelligencer, will show the estimation in which it is held.

"We are not aware that this number, whose contents we have briefly sketched, is richer than usual. We believe it steadily supports the character it has earned, of being, what good authorities have pronounced, the best scientific periodical in the English language which covers an equal, or nearly equal field. Its editorial staff warrants its abilities. Besides its venerable founder, there is the younger Silliman, and Professor J. D. Dana, our most encyclopædic and ablest general naturalist, assisted by Agassiz, Gray, and Wolcott Gibbs, in their several departments."

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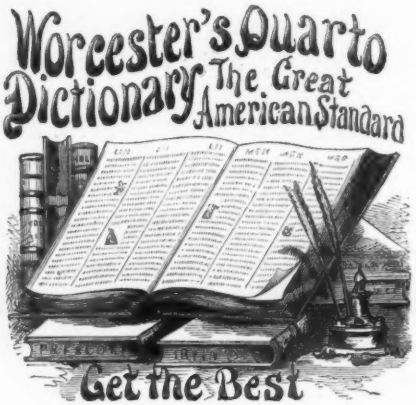

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